Reference systemso

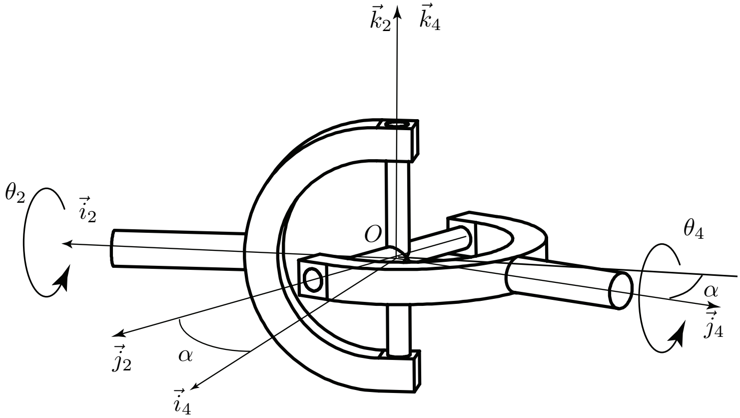
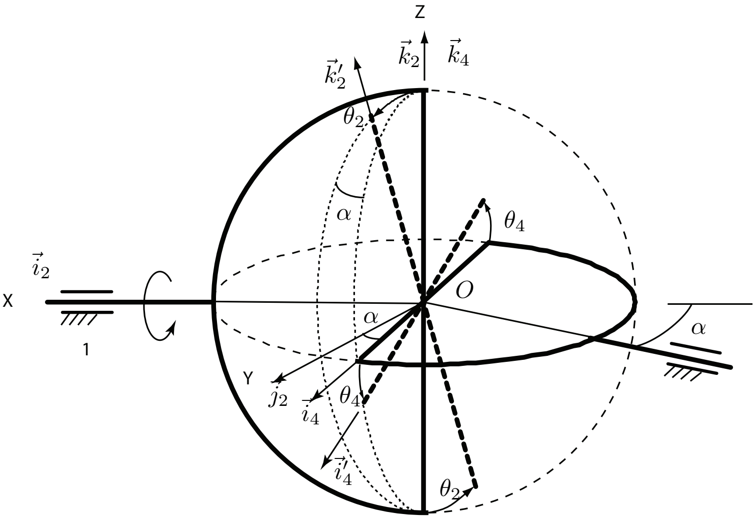


Figura : Cardan joint


Z
$\vec{k}_2$
$\vec{k}_4$
$\vec{k}'_2$
θ_2
α
θ_4
$\vec{i}_2$
X
O
α
α
1
Y
$\vec{j}_2$
$\vec{i}_4$
θ_4
$\vec{i}'_4$
θ_2

The Cartesian components of $\vec{k}_2'$ e $\vec{i}_4'$ in $O - XYZ$ are, respectively,

$$\vec{k}_2' \equiv \left\{ \begin{array}{ccc} 0 & \sin\theta_2 & \cos\theta_2 \end{array} \right\} ,$$
$$\vec{i}_4' \equiv \left\{ \begin{array}{ccc} 0 & \cos\theta_4 \cos\alpha & -\sin\theta_4 \end{array} \right\} .$$

These unit vectors will be always orthogonal:

$$\vec{k}_2' \cdot \vec{i}_4' = \left\{ \begin{array}{ccc} 0 & \sin\theta_2 & \cos\theta_2 \end{array} \right\} \left\{ \begin{array}{c} 0 \\ \cos\theta_4 \cos\alpha \\ -\sin\theta_4 \end{array} \right\}$$
$$= \sin\theta_2 \cos\theta_4 \cos\alpha - \cos\theta_2 \sin\theta_4 = 0 ,$$

or

$$\tan\theta_2 \cos\alpha = \tan\theta_4 . \qquad (1)$$

Differentiating with respect to time we have

$$(1 + \tan^2 \vartheta_4)\dot{\vartheta}_4 = \cos\alpha(1 + \tan^2 \vartheta_2)\dot{\vartheta}_2 \ ,$$

Transmission ratio

$$\tau = \frac{\dot{\vartheta}_4}{\dot{\vartheta}_2} = \frac{\cos\alpha(1 + \tan^2 \vartheta_2)}{(1 + \tan^2 \vartheta_4)} \ ,$$